Mathematical Competence Exhibited by Year 2 Students When Learning Through Sequences of Challenging Tasks

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Developing reliable and effective processes to monitor and interpret student progress during problem-solving tasks is an ongoing challenge in mathematics education. This study utilised qualitative data sources inclusive of observations, work samples and interview transcripts from six Year 2 students to investigate exhibited competence in classroom settings. The analysis showed that students demonstrated growth in both cognitive and dispositional elements of competence when learning through sequences of challenging tasks. These elements, consistently identified across different class settings and lesson topics, have the potential to broaden interpretations of mathematical competence within both practice and research domains.

An ongoing challenge within mathematics education is being able to identify what it means to become mathematically competent. Interpretations of competence are heavily context-dependent, leaving little consensus as to how the construct should be defined (Ropohl et al., 2018). Within the discipline of mathematics, definitions of competence become particularly relevant when considering the most effective ways to teach. For example, those supporting traditional approaches to mathematics learning emphasising the mastery of skills and procedures before problem-solving (e.g., Kirschner et al., 2006), are likely to consider competence as the demonstration of such processes. Alternatively, those aligned with reformist orientations, recognising the collaborative nature of problem-solving pedagogies, lean towards more holistic interpretations of competence (Blömeke et al., 2015). From this perspective, value is placed on the simultaneous development of students' conceptual understanding and positive dispositions within mathematics learning. However, a prominent barrier to the latter interpretation of competence is being able to accurately identify and measure the complex way such elements of students' learning manifest and develop.

The research presented in this paper aligns with the interpretation of competence presented by Blömeke et al. (2015), positioning mathematics learning within a challenging task approach (Sullivan et al., 2015). Learning mathematics through a challenging task approach creates opportunities for students of all abilities to engage with cognitively demanding, non-routine problems by: utilising prior knowledge; exploring multiple solutions; and working collaboratively to deepen conceptual understanding (Sullivan et al., 2020). Previous research on challenging tasks has focused on teacher professional development (e.g., Ingram et al., 2020; Sullivan et al., 2015) and student achievement in the middle years (e.g., Sullivan et al., 2016) with limited focus on students in the Early Years (e.g., Hubbard et al., 2022; Russo & Hopkins, 2017). Therefore, identifying how Year 2 students demonstrate and develop mathematical competence when learning through challenging tasks will contribute to the literature. Specifically, this paper aims to address the following research question:

- How do Year 2 students demonstrate and develop mathematical competence when learning through sequences of connected, cumulative and challenging tasks?

Literature Review

Measures of student achievement in mathematics have traditionally relied on tests and written assessments to evaluate student learning. A constraint of these practices is that the fundamentals of mathematics learning are often misrepresented (Clarke, 2011). Broadening the scope of assessment practices to more accurately reflect students' mathematics learning experiences continue to receive

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attention within the literature. Examples of alternative forms of mathematics assessment designed to address this gap have included: multi-mode interviews (Kuzle, 2017); comparative judgement of rich assessment tasks (Jones et al., 2015); and the development of marking keys for interpreting student responses to sequences of non-routine tasks (Hubbard et al., 2022). These studies reported on processes deemed effective in evaluating student competence more comprehensively than traditional tests by enabling the demonstration of both core skills and higher-order thinking. However, the corresponding assessment tasks were disconnected from students' actual classroom experiences, continuing to frame achievement as individual cognitive performance.

Observations of students' mathematical learning in classrooms have the potential to provide insights into competence that cannot be ascertained otherwise, yet the abundance of qualitative sources can become problematic in clarifying the construct of competence. Ropohl et al. (2018) identified one of these challenges as being able to make sense of the complexities within the data. Schlesinger and Jentsch (2016) conducted a systematic literature review focusing on the methodologies used to carry out such classroom observations. The authors reported that little consistency exists in identifying specific aspects of competence being studied and that the unique classroom settings pose challenges in making generalisations from the findings. For example, while Özdemir and Pape (2012) conducted a four-month study revealing that specific classroom practices enhanced students' ability to self-regulate their learning in mathematics, the findings were specific to one Year 6 class, restricting the transferability to other contexts. Another concern raised by Schlesinger and Jentsch (2016) was validity issues that occur when data collection relies too heavily on the recollections of participants. Boesen et al. (2014) encountered this issue when working with over 200 teachers for 12 months in a professional learning capacity. Their study found that despite clearly articulating diverse notions of mathematical competence throughout their program, the participants' final reflections of student achievement were dominated by traditional perceptions of ability such as the accurate completion of procedural tasks. The tendency to preference cognitive achievement over dispositional elements of student learning is not uncommon when defining mathematical competence and one that Beyers (2011) attributed to limited research on the influence dispositions have on mathematical thinking.

Blömeke et al. (2015) suggested adopting analytical processes that recognise related observable behaviour and cognitive abilities in preference to approaches that isolate discrete elements of competence. In doing so, "the successful deployment of capabilities in engagement with mathematical problems and the language and tools of mathematics" (Ropohl et al., 2018, p. 17) can be identified and evaluated. Chan and Clarke (2017) demonstrated this approach in an analysis of video data and student work samples of two Year 7 classes working collaboratively on problem-solving tasks. Their findings showed that the observed negotiations and interactions of students, while rich and complex, could be analysed within several themes comprising both mathematical and social, enabling consistent analysis across the different classes. Similarly, Groth (2017) introduced a proficiency protocol to guide the observation of prospective teachers across a series of lessons based on multiple Year 7 mathematics classes. The provision of the protocol supported a focus on the dispositional aspects of student learning, leading to a better collective understanding of the nuanced happenings occurring across the different mathematics lessons. Adopting similar processes could provide further insights into the ways students holistically develop mathematical competence when learning through sequences of challenging tasks in the Early Years.

Methodology

This study was conducted within a larger research project entitled *Exploring Mathematical Sequences of Connected, Cumulative and Challenging Tasks* (EMC^3) (Sullivan et al., 2020). Building upon previous work on challenging tasks (see Sullivan et al., 2015), EMC^3 focused on the ways sequences of challenging tasks and the associated pedagogies support mathematics learning for students in Foundation to Year 2 (5- to 8-years old). One outcome of the EMC[3] project to date was the development of an instructional model supporting the implementation of tasks and ensuring adequate provision of agency and inclusion for students (Sullivan et al., 2021). The model, based on the work of Smith and Stein (2011), encourages teachers to anticipate student responses before planning lessons within three structured phases: *Launch, Explore* and *Summarise/Review*.

The focus of this study was to investigate, the ways Year 2 students demonstrate and develop mathematical competence when learning through sequences of challenging tasks. Six focus students (two students across three classes) were selected from a total of 59 Year 2 students at one of the participating EMC[3] project schools. Students were selected by identifying prior mathematical achievement with the intention that the focus students would represent the diversity of the overall Year 2 cohort. Fred, Jess, and Tim demonstrated moderate levels of achievement while Zara, Evie, and Annie (all pseudonyms) represented students with lower mathematical levels of achievement. Qualitative data were collected to create *learning portfolios* intended to track changes in student learning as they participated in the EMC[3] project. Lesson notes were collected using an observation protocol based on the phases of the EMC[3] instructional model. These notes along with student work samples and post lesson interview transcripts were collated from the first three consecutive lessons of the study (Portfolio 1) and compared to the same data sources collected from three consecutive lessons nine-months later (Portfolio 2).

Data Analysis

Lesson artefacts were created to identify the cognitive and dispositional behaviours students demonstrated for each lesson. Figure 1 presents two of Annie's lesson artefacts taken from Portfolio 1 and 2 respectively.

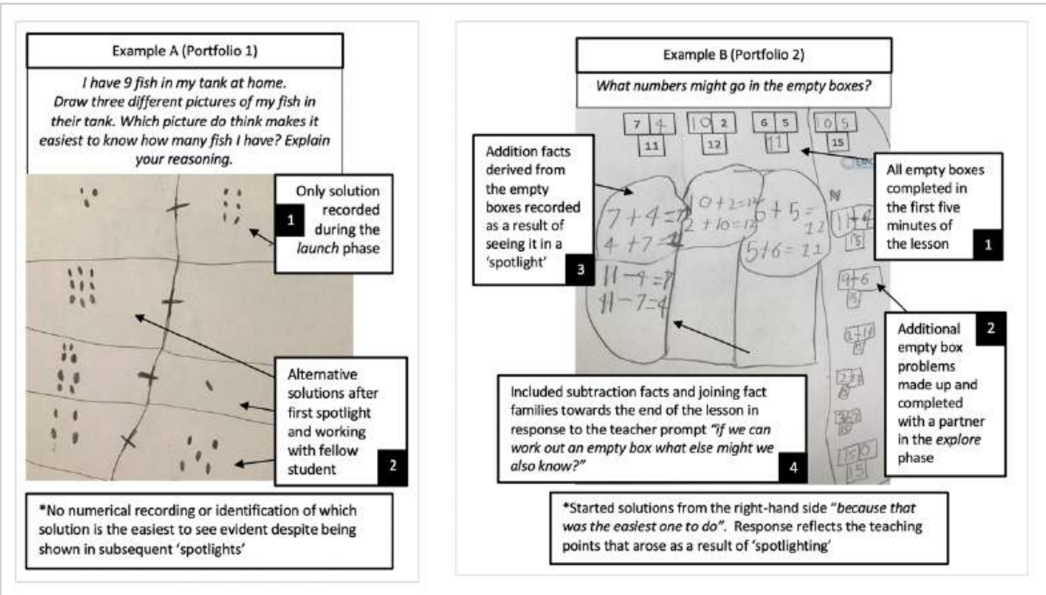

Figure 1. Comparative lesson artefacts (Fish Tank task & Empty Boxes tasks).

The process of annotating work samples with time bound lesson observations was based on a similar method documented by Schoenfeld (2016). This holistic interpretation of student learning highlighted cognitive and dispositional behaviours students exhibited throughout the study, enabling competence elements that aligned with the three EMC[3] lesson phases to be generated. The presence of each competence element evident in the artefacts was then coded according to the following: (√) *Most of the time* (three examples, one from each lesson); (♦) *some of the time* (1 to 2 examples, over any lesson); and (-) *not evident* (no example). The coding enabled comparisons to be made between the extent that these elements were present in Portfolios 1 and 2 and were reported in the results.

Results

The results report on the coded competence elements according to the three EMC^3 lesson phases: *Launch, Explore, Summarise.* The first line of coding for each competence element corresponds to Portfolio 1 whereas the second line of coding represents the presence throughout Portfolio 2 (see Tables 1, 2 & 3).

The Launch Phase

The competence elements identified within the *Launch* phase are presented in Table 1. As part of this phase, the task is posed to students without explicit instruction and students are provided with approximately five minutes to engage with the task independently.

Table 1

Competence Elements Demonstrated Throughout the Launch Phase of the Lesson

Competence elements *Launch* phase	Fred	Jess	Tim	Zara	Evie	Annie
Demonstrates a willingness to independently read and attempt task without further instruction	♦	♦	√	√	√	♦
	√	√	√	√	√	√
Records mathematics related to the task	♦	♦	√	♦	♦	♦
	√	√	√	√	√	√
Provides more than one correct solution while working independently	♦	♦	♦	-	-	-
	√	√	√	√	√	√
Demonstrates an awareness of connections to prior learning	♦	♦	♦	♦	♦	♦
	√	√	√	√	√	√

Table 1 shows throughout Portfolio 2 data, students consistently demonstrated each of the necessary elements that comprise competence, regardless of their prior mathematical achievement status. On its own, the first element, *a willingness to independently read and attempt the task without further instruction*, could simply be representative of compliant and well-behaved students. However, when taken with the presence of the additional three elements (*demonstrating initial mathematics thinking; representing multiple solutions; connecting knowledge to previous experiences*) their willingness to engage independently is likely to be indicative of productive dispositions. The interconnectedness of these elements can be illustrated through the lesson artefacts in Figure 1. Annotations in Example B show that in the first five minutes of the lesson, Annie was able to provide multiple solutions to the four empty box tasks and these solutions were reflective of mathematical thinking (Figure 1, Example B, annotation 1). Observation notes recorded at the bottom of Example B shows that an additional factor was the utilisation of prior knowledge, as Annie started with the empty boxes on the right-hand side *'because it was the easiest one to do'* (Annie, interview transcript). Contrast these elements with the annotations from Example A detailing less evidence of mathematical thinking or the utilisation of prior learning, and the way Annie willingly and independently engaged with the tasks in the *Launch* phase by Portfolio 2 becomes apparent.

The Explore Phase

The *Explore* phase of the lesson opens up the learning experience to encourage students to share their initial thinking and begin collaborating with peers. In this study, teachers selected specific work samples to show the class initiating student-centred discussions. This technique was referred to as 'spotlighting'. Table 2 presents the competence elements identified throughout the *Explore* phase.

Table 2

Competence Elements Demonstrated Throughout the Explore Phase of the Lesson

Competence elements *Explore* phase	Fred	Jess	Tim	Zara	Evie	Annie
Attentive during spotlighting (i.e., paying attention to speaker, focused on work being shared)	♦ ♦	♦ ♦	♦ ♦	♦ √	♦ √	♦ √
Responsive to spotlighting (i.e., changing strategy or working out after seeing other solutions)	♦ √	♦ √	♦ √	♦ √	♦ √	♦ √
Collaborates with peers	♦ √	♦ √	♦ √	- √	- √	- √
Responds productively to feedback	- √	- √	- √	- √	- √	- √
Work sample demonstrates multiple solutions or solution pathways	♦ √	♦ √	♦ √	♦ √	♦ √	♦ √

The changes presented in Table 2 show an overall increase in teacher and peer interaction by the end of the study. Warranting further attention is the degree to which some of these elements improved in comparison to others. Extensive development is evident in the way students responded to feedback. The initial lesson artefacts from Portfolio 1 show a range of unproductive habits such as students rubbing out their work or covering their page when asked a question. The following excerpt from an interaction documented at the start of the study demonstrates an example of a less productive response to feedback:

Teacher: Can you tell me what you have done here? (Pointing to 6 dots arranged in 2 rows of 3)

Fred: This is a solution to how the fish would be arranged.

T: I am wondering if this shows all the fish (and points back to the written task where 9 fish is recorded).

F: Oh, it is a mistake (student proceeds to get the rubber and rub out the solution).

In this exchange, Fred's actions suggested that incorrect solutions should not be included within a work sample and needed to be rubbed out and removed. The interaction suggests that Fred was not used to correcting existing solutions (which would have been possible by adding another row of dots) implying that he considered the teachers' role was to correct work rather than support the development of his learning. Lesson observations from the end of the study show more productive responses to feedback opportunities. For example, one student volunteered that *'this part of the page shows my thinking at the start'* (Evie) and another articulated *'I have added in these labels to show what the diagram means'* (Tim). These shifts suggest that the use of guiding and clarifying questions to facilitate interactions between teachers and students likely changed the perception that feedback was intended to support their learning, rather than evaluate it.

The role of 'spotlighting' became a critical element in supporting students to develop competence within the *Explore* phase. How students responded to 'spotlighting' was evident in the ways students explained their written responses. For example, Jess indicated that *'I have done the same strategy but set it out in a table so I will just keep going my way'* when asked if a previous 'spotlight' helped her with the solution. That students can discern if it is necessary to make changes to their thinking is an encouraging observation, even when they were ostensibly not attending to the spotlight discussion. This suggests students have developed an understanding that the purpose of a 'spotlight' is exposure to alternative thinking, not necessarily to showcase a preferred solution or procedure.

The Summarise Phase

The competence elements identified throughout the final phase of the lesson are presented in Table 3. The *Summarise* phase provides an opportunity for the whole class to review the mathematical focus of the lesson and discuss their learning experiences.

Table 3

Competence Elements Demonstrated Throughout the Summarise Phase of the Lesson

Competence elements Summarise phase	Fred	Jess	Tim	Zara	Evie	Annie
Demonstrates attentive behaviours during class discussion	♦	♦	♦	♦	♦	♦
	♦	♦	♦	√	√	√
Willingness to contribute to class discussions	♦	♦	♦	-	-	-
	√	♦	√	♦	√	√
Written response reflects thinking trajectory over the whole lesson	♦	♦	♦	-	-	♦
	√	√	√	√	√	√

Table 3 shows that by the end of the study, students demonstrated competence to varying degrees throughout the *Summarise* phase. The element that represented the greatest increase was students' written responses, better reflecting student thinking over the whole lesson. The lesson artefacts from the end of the study not only provide broader examples of strategies and solutions but also indicate more flexibility and connectivity of mathematical concepts. Being able to produce a written response in this manner shows students recognised their work reflects learning and understanding rather than an outcome derived from mimicking given processes.

Discussion and Conclusion

The findings of this study support the claim of Ropohl et al. (2018) that observations within classroom environments afford greater insights into the holistic notion of mathematical competence than can be determined from a written assessment or single work sample analysis. Triangulating the qualitative data sources to create lesson artefacts representative of both cognitive and dispositional behaviours as recommended by Blömeke et al. (2015), extended the interpretation of work samples beyond traditional outcomes and better represented the mathematics problem-solving experiences of students (Clarke, 2011). Using the three lesson phases from the EMC³ project to structure observations proved beneficial in terms of ensuring the nuanced differences in student learning, both within a complete lesson and over a series of lessons, could be accurately identified and compared. Similar to the findings from Chan and Clarke (2017), being able to analyse classroom observations through such schema enabled consistency in the interpretation of student competence across multiple classes. Moreover, the competence elements identified in the different lesson phases showed transferability across class contexts and different lesson topics, reinforcing the notion that competence is broader than lesson-specific content and procedures.

As well as emphasising the interconnectedness of cognitive and dispositional components of mathematics learning, the findings reported various ways sequences of challenging tasks support Early Years students to experience success with their learning. Being able to specify when and how students access their prior knowledge, engage with alternative solutions and respond to feedback encapsulated the experiences of mathematics learning as intended through the EMC³ approach (Sullivan et al., 2020). Furthermore, the growth demonstrated by the focus students, regardless of their prior achievement status suggests that a broad range of students can effectively develop mathematical competence when learning through the EMC³ instructional model (Sullivan et al., 2021). Particularly insightful was the impact that communication, both between peers and student/teacher interactions had on strengthening student competence, and mirrored the findings from Chan and Clarke (2017) that social components are central to successful mathematics learning. Recognising that these elements are critical in mathematics development, even for students in the Early Years, may help teachers to shift the emphasis away from traditional measures of competence and more accurately target suitable areas for future learning. While limitations of this study include a small sample size within a single school setting, the competence elements identified through this investigation may provide a useful structure to guide further research in this area.

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