Planning and Anticipating Early Years Students' Mathematical Responses

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This paper reports on early years teachers and how often they should devote planning time to anticipating student responses in advance of the lesson. Sixty-five Foundation to Year 2 teachers (students 5-8 years of age) completed questionnaires at the beginning and end of a year-long research-based professional development program. Participants were learning to teach with sequences of challenging tasks. Post-program data showed a shift in the frequency of time participants believed teachers should devote to anticipating student responses prior to teaching. Supporting teachers' mathematical knowledge for teaching with an emphasis on how they plan and anticipate student responses has implications for improving practice and student outcomes.

In preparation for teaching mathematics lessons, it is recommended that teachers anticipate student responses before the lesson. This practice is crucial as it allows teachers to consider what students might do and how they might respond during the lesson (Smith et al., 2020).

> Through planning, teachers can anticipate likely student contributions, prepare responses that they might make to them, and make decisions, about how to structure students' presentations to further their mathematical agenda for the lesson. (Smith & Stein, 2018. p. 9)

Smith and colleagues recommend that all mathematics teachers develop the practice of anticipating. Yet some teachers who teach Foundation to Year 2 students (5-8 years of age) may have a belief that they do not feel the need to anticipate student responses when planning. They may assume the "mathematics is easy" and they should be able to respond in the moment of teaching without detailed planning.

In our research project Mathematics Sequences of Learning (MSoL), we were interested in identifying if and why early years teachers when teaching sequences of challenging problems (tasks) devote planning time to anticipating student responses in advance of teaching. We sought to respond to the following research questions:

- How often should early years teachers devote planning time to anticipating student responses?
- How do early years teachers explain the practice of anticipating when planning sequences of challenging tasks?

Literature

Theoretical Framework

Mathematical Knowledge for Teaching (MKT) is informed by subject matter knowledge and pedagogical content knowledge (PCK) (Ball et al., 2008). PCK is a special kind of knowledge that is unique to teachers. When classifying pedagogical content knowledge (PCK), Ball et al. describe three domains, knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum. When demonstrating PCK teachers know what and how to teach a topic in combination with knowledge of instruction, activities, and evaluation tools (Brophy, 1991). Equally, MKT will influence what and how teachers plan for teaching.

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Planning for Mathematics Teaching

Typically, primary school teachers plan with colleagues and a mathematics leader. These colleagues are often middle-school leaders who specialised in mathematics teaching and are responsible for leading team meetings (Davidson, 2019). Helping teachers consider what they can do prior to teaching has the potential to guide student thinking and discussion as well as mathematical instruction (Stein et al., 2008). Others agree that planning for mathematics teaching is complex (Davidson, 2019; Smith & Stein, 2018; Vale et al., 2019) and should be supported by sharing lesson ideas as a collaborative experience that teachers do together (Ebaeguin & Stephen, 2016). Depending on the size of the school approaches to planning for teaching may vary. In Australian schools, teachers are usually given one hour per week to plan mathematics lessons (Davidson, 2019). They might research curriculum materials and prepare detailed lesson plans (Ebaeguin & Stephen, 2016) filling in a planning proforma (e.g., Smith & Stein, 2018) that caters for different student learning needs.

Differentiating instruction includes teaching practices to support the needs of all students (Tomlinson, 2014). When planning, tasks can be chosen by teachers because of their differing potential (Bardy et al., 2021). Such as, sequences of challenging tasks have been reported to support differentiated learning experiences more effectively than other pedagogical approaches, although this is contingent on the teacher playing an active role to contextualise tasks, using open prompting questions, and facilitating sharing of student work (Russo & Hubbard, 2022). Such actions are supported by access to professional reading allowing teachers to extend their knowledge of theory and practice by identifying key mathematical concepts, ideas, skills, and language for each lesson (Davidson, 2019). Other recommendations when planning for differentiation includes the identification of common misconceptions, anticipating all possible solutions as well as likely student strategies (Smith & Stein, 2018).

Anticipating Student Solutions

When reflecting on teaching problem-solving with secondary students, Wallace (2007) was concerned that rather than "problem-solving" her students were trying to solve the problem (task) the way she wanted. Not wanting to take over students' thinking, when planning lessons, she began to anticipate both correct and incorrect responses to problems. By anticipating how students might respond to a problem Wallace believed this allowed her to think ahead during the lesson and assisted her questioning for guiding learning.

Teachers should be aware of the importance of planning with others to maximise opportunities for anticipating a range of possible solutions and strategies for solving a task before teaching. When planning with colleagues some teachers might be surprised at the possible number of solutions to a task. For example, a Year 3 teaching team identified 16 anticipated student solutions which were consequently produced by the students during the lesson (Vale et al., 2019). In addition, Stein et al., (2008) suggest that when students solve student-centred instructional tasks, they will solve them in more than one way. In summary, there is agreement that teachers first need to solve the problem themselves prior to teaching (Sullivan et al., 2015). Specifically, anticipating student solutions should include identifying a range of solutions/and or strategies (Smith & Stein, 2018). Others suggest the anticipated solutions can then be ordered as a trajectory of learning that can be considered by the teacher to scaffold student learning during the lesson (Vale et al., 2019).

Acknowledging that students will respond to tasks with a range of different answers, Smith & Stein (2018) designed a model to support teachers when considering ways to guide mathematical discussion during the lesson. They developed a model that included five practices: *Anticipating. Monitoring, Selecting, Sequencing* & *Connecting* for supporting lesson planning protocols. The teachers participating in the MSoL study were introduced to the five practices (Smith & Stein, 2018) and provided with research-informed sequences of lessons (Sullivan et al., 2023) and an instructional model (that included anticipating) for teaching challenging tasks (Bobis et al., 2021). By doing so we hypothesised that teachers would dedicate more planning time to anticipating student responses.

Method

Context and Participants

Participants included Foundation Year to Year 2 teachers and mathematics leaders (N=96) from 19 Catholic Primary Schools in Australia. They were participating in a year-long research-designed professional learning program MSoL related to teaching with sequences of challenging tasks (Sullivan et al., 2023). The aim of the program was to extend teachers' MKT when implementing research-designed resources 'Exploring Mathematical Sequences of Connected, Cumulative and Challenging Tasks' (EMC^3) (LP180100611). Fifteen sequences of lessons (provided in a resource book) and an instructional model (Bobis et al., 2021) were designed to support teachers to build new understandings of student-centred approaches to teaching mathematics in the early years.

The participants attended three professional learning days (April-July-October) with the research team (authors). Most participants were familiar with teaching challenging tasks, but not the resources provided as part of their professional learning. Three days of professional learning were designed to support teachers' PCK and provided strategies for planning and implementing the sequences of lessons.

Professional development can be effective when it focuses on situations in practice (e.g., Lipowsky & Rzejak, 2015). Therefore, Day 1 introduced teachers to the student-centred inquiry approach. Day 2 included a session on the '*5 Practices for Orchestrating Productive Mathematics Discussions*,' and planning approaches (Smith & Stein, 2018) and questioning strategies (Livy et al., 2021). Day 3 focused on assessment practices and rubrics (Hubbard et al., 2022). In addition, two different sequences were introduced to the teachers for each day of professional learning. This included anticipating possible solutions and strategies students might choose, including misconceptions or partial conceptions, when solving the task.

Data Collection

Sixty-five participants responded to a 20-minute pre-program and post-program online Qualtrics questionnaire at the beginning of Days 1 and 3. In educational settings, online questionnaires are often used as a method of data collection because they have a better response rate and are more reliable when compared to pen-and-paper surveys (Seleh & Bista, 2017). The 16-item questionnaire included six demographic items and five Likert-style items each followed by an open question asking teachers to explain their responses to the previous Likert-style item. Two items (Q15 and Q16) are reported in this paper.

For the four-point Likert-style item Q15 pre- and post-questionnaire participants were asked, "When planning for teaching with challenging tasks should Foundation to Year 2 teachers devote planning time to anticipating student responses in advance of the lesson?" and selected *never*, *sometimes*, mostly, or *always.* Since we were particularly interested in participants' responses after the intervention, we have only reported on post-questionnaire responses to the question (Q16), "Explain why you think this?"

Data Analysis and Coding

For Q15, data were entered into a statistical software suite (SPSS). A Wilcoxon signed-rank test was used to compare the data of the pre- and post-questionnaire responses to determine if the two samples showed a statistically significant change. The results and analysis included a report of the frequency of responses to four-point Likert-style responses of *never*, *sometimes*, mostly, and *always* (see Table 2 results). For Q16, participants' responses were entered into an excel spreadsheet for coding. The first two authors independently used open coding, they met to discuss, collate codes, and then agreed on six codes as shown in Table 1. Longer responses were coded using two or three codes. Codes were checked by the third author.

Table 1

Open Coding Categories

	Code	Description of code	Example from text
1	Differentiation of student knowledge	Prepare prompts; plan for responses; cater for everyone	Create enabling and extending prompts
2	Misconceptions	Identifying misconceptions	Know what the misconception might be
3	Orchestrating discussion	Planning questions; when to discuss student responses; anticipating responses	It helps to have knowledge so that you can so you can talk about strategies
4	Scaffolding the lesson	Guiding learning; considering how to adapt the lesson; selecting student work samples to share	Being prepared for their responses can help us come up with ways to guide students
5	Time constraints	Reference to time as a barrier when planning	Limited planning time
6	Other	The response did not match previous codes	It's good to keep an open mind

The frequency of responses to Q15 was tallied and converted to percentages (see, Table 2). For Q16 the number of responses for each code was tallied. Next the percentage of participants whose responses matched each code was calculated (see, Table 3).

Results and Discussion

The results and discussion include a comparison of both pre-program and post-program Likert-style items (Q15) followed by the results and discussion of the open-response item (Q16) collected at the end of the program.

Frequency of Planning Anticipated Responses

For Q15 the comparison of pre-program and post-program responses reveals differences between teachers' rankings ranging from 'Never' (1) to 'Always' (4) (Table 2). The results in Table 2 show a shift from 'Sometimes and Mostly' to 'Mostly and Always' when comparing the percentage of participants' responses pre-program and post-program. A total of 60% of participants shared the belief that teachers should either sometimes or mostly anticipate student responses before teaching. After participating in the project and extending their knowledge for planning and anticipating student solutions and strategies 86% of teachers agreed anticipating should occur mostly or always.

Table 2

Q15 Pre- Post-frequency 14 (N=65)

Response	Pre-course frequency (%)	Post-course frequency (%)
1. Never	3%	0%
2. Sometimes (i.e., in a minority of lessons when teaching with challenging tasks)	20%	14%
3. Mostly (i.e., most lessons when teaching with challenging tasks)	40%	34%
4. Always (i.e., every lesson when teaching with challenging tasks)	37%	52%

In addition, a statistical analysis of these results compared the mean ranking of the pre- and post-program questionnaire data using a Wilcoxon signed-rank test. The results showed that participating in the professional learning program elicited a statistically significant change when considering the importance of devoting planning time to anticipating student responses in advance of the lesson ($Z = 2.660$, $p = 0.008$). Indeed, the median rating shifted from 4.00 as something teachers should mostly do when teaching challenging tasks in the early years of primary school to 5.00 as something they should always do.

The next section reports and discusses the participants' explanations of their response to planning time and anticipating student responses.

Reasons for Planning Anticipated Responses

For Q16 the participants were asked to explain their responses to Q15 (Table 3).

Table 3

Frequency of Codes Per Participant to Question 15 (Devoting Planning Time) Post Program

	Code	Frequency of responses (n=89)	Percentage of participants (n=65)
1	Differentiation of student knowledge	31	48%
2	Misconceptions	9	14%
3	Orchestrating discussion	10	16%
4	Scaffolding the lesson	26	40%
5	Time constraints	9	14%
6	Other	4	6%

Table 3 includes the six codes used to code all participant responses and the frequency of responses (n=89) for all codes. The final column is the percentage of codes for the number of participants (n=65) and the total percentage is greater than 100 percent because sometimes more than one code occurred in one teacher’s response. For example:

> When you plan for student responses, it allows you to scaffold their learning and plan for tasks based on the student's learning ability. (Codes 1 and 4)

The most common explanations related to Code 1 ‘differentiation of student knowledge’ and Code 4 ‘scaffolding the lesson.’

Differentiation of Student Knowledge

Nearly half (48%) of the participants' responses were coded as Code 1 highlighting teachers' acknowledgement of the importance of differentiation of student knowledge when anticipating. For example, one teacher considered the difficulty of the task, and another acknowledged the importance of considering the range of answers:

> To determine the level of difficulty and whether the students will be able to understand, [or] gain anything from the task. (Codes 1 and 4)

> To support their learning and understand their thinking. (Code 1)

Many of the other responses referred to supporting learning including the planning of *enabling* or *extending prompts* to use during the lesson. Enabling prompts are designed for students experiencing difficulties and extending prompts are designed for students who may complete the main task quickly (Sullivan et al., 2015). Participants were introduced to this practice of supporting student differentiation by using these prompts.

Scaffolding the Lesson

More than one-third (40%) of participants' responses were coded as Code 4 and made connections with scaffolding the lesson. One teacher revealed, "It helps to have an idea of where the lesson might go and to be prepared for that." Other examples include, "to know what the next step is" as well as "to direct how the lesson will go."

These responses indicated the teachers' developing understanding of scaffolding learning by posing questions designed to extend student learning. However, it is important not to take over student thinking but for teachers to guide learning (Wallace, 2007).

Whereas another teacher mentioned how the school leader supported planning.

> I especially like doing the task together as a year level to guide us in anticipating student responses. (Codes 1 and 4)

This response confirms the importance of collaborative planning. Davidson, (2019) would agree that collaborative experiences will support teaching practices.

In summary, Codes 1 and 4 show evidence of PCK because these teachers were making connections with anticipating and differentiating or scaffolding the lesson, which could support both their KCS and KCT for teaching.

Code 5: Time Constraints

Some responses (14%) focused on time constraints as a barrier. This should be addressed by school leaders and schools to ensure teachers have time to plan, especially when learning new approaches for teaching and planning. Examples of time constraints included:

> Time is restricting in planning, so it would be impossible to do it for each challenging task. (Code 5)

Another reason for not being able to plan for every lesson included the time needed for the detail expected when planning and anticipating student responses for each lesson:

> The planning process is very thorough and would be difficult to fit in with other curriculum requirements. (Code 5)

Whereas one teacher wrote about the importance of finding time to plan:

> It would be amazing to have the time to do it for all maths planning sessions however it is not realistic. It is so important to make the time if possible to prepare how you would respond. (Code 5)

Time constraints as barriers were because the teachers were not provided enough time to plan the detail they desired and needed prior to teaching a sequence of lessons. These findings were not surprising given that typically teachers are provided with one hour for planning mathematics lessons each week in primary schools (Davidson, 2019).

These results may have been influenced by different factors including participation across the three days of the intervention. Although not reported, perhaps teachers previously devoted time to sourcing tasks for teaching, limiting time for anticipating student responses. A benefit of using the EMC^3 resources and lesson approach they experienced during the intervention suggests that teachers were able to shift their thinking related to anticipating prior to teaching. Changing practices when planning could assist teachers to improve their MKT by developing an awareness of different ways to solve tasks. To support the practice of anticipating we recommended that teachers plan collaboratively to ensure they identify all possible solutions and think deeply about what students are likely to do and how they will approach the problem (Smith et al., 2020).

Conclusion

As part of the MSoL project, we were interested in supporting teachers' PCK including when planning to teach with sequences of challenging tasks. Prior knowledge of teaching challenging tasks may have influenced these results. When participating in professional learning, prior knowledge can either assist teachers to extend their knowledge or hinder learning (Lipowsky & Rzujak, 2015). The findings of our study suggest that the program extended participants' PCK about planning and anticipating student responses because in the post-questionnaire three-quarters of Early Years teachers from 19 different schools reported that teachers should mostly or always anticipate student responses before teaching. When explaining the practices of anticipating they focused more on approaches for differentiation of student knowledge and scaffolding learning. Other approaches included a focus on the identification of possible misconceptions and orchestrating discussions. Each of these responses suggests enhancement of teachers' PCK including KCS and KCT.

A barrier for some teachers included time constraints. Not stated by the participants of this study but an implication worth noting for teachers is that they would have more time for anticipating student responses to the task because they were provided with fourteen sequences of learning experiences (challenging tasks) (Sullivan et al., 2023). As a result, the project teachers had more time during their planning to extend their knowledge for teaching by anticipating student responses. Such practices suggest these teachers can be better prepared for teaching than in the past. It would be expected as teachers become more familiar with the tasks, anticipating student responses and planning teacher actions will be less demanding the following year if teachers consolidate their teaching with the same year level. However, teachers may begin to plan their own sequences as they become familiar with the EMC^3 approach, which would take time away from anticipating student responses. In summary supporting teachers' MKT with an emphasis on how they plan and anticipate student responses has implications for supporting student learning and approaches for improving teachers' KCS and KCT. This study was limited by its focus on self-reported teacher data. Further research including longitudinal data, observing teachers' planning and follow-up interviews are needed to extend the current study's findings.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5)*,* 389–407. https://doi.org/10.1177/0022487108324554

Bardy, T., Holzäpfel, L., & Leuders, T. (2021). Adaptive tasks as a differentiation strategy in the mathematics classroom: Features from research and teachers' views. *Mathematics Teacher Education and Development, 23*(3), 26–53.

Bobis, J., Russo, J., Downton, A., Feng, M., Livy, S., McCormick, M., & Sullivan, P. (2021). Instructional moves that Increase chances of engaging all students in learning mathematics. *Mathematics*, *9*(6), 582. https://doi.org/https://doi.org/10.3390/math9060582

Brophy, J. E. (1991). Conclusion to advances in research on teaching: Teachers' knowledge of subject matter as it relates to their teaching practice. In J. E. Brophy (Ed.), *Advances in research on reaching researchers' subject matter knowledge and classroom instruction* (Vol. 2, pp. 347–362). JAL Press.

Davidson, A. (2019). Ingredients for planning student-centred learning in mathematics. *Australian Primary Mathematics Classroom*, *24*(3), 8–14.

Ebaeguin, M., & Stephens, M. (2016). Key elements of a good mathematics lesson as seen by Japanese junior high school teachers. In B. White, M. Chinnappan, and S. Trenholm (Eds.), *Opening up mathematics education research. Proceeding of the 39th annual conference of the Mathematics Education Research Group of Australasia (pp. 206–213).* MERGA

Hubbard, J., Russo, J., & Livy, S. (2022). Assessing mathematical competence through challenging tasks. In N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys. Proceedings of the 44th annual conference of the Mathematics Education Research Group of Australasia* (pp. 282–289). Launceston: MERGA.

Lipowsky, F., & Rzejak, D. (2015). Key features of effective professional development programmes for teachers. *Ricercazione, 7*(2), 27–51.

Livy, S., Bobis, J., Downton, A., McCormick, M., Russo, J., & Sullivan, P. (2021). Teacher actions for consolidating learning in the early years. In Y. H. Leong, B. Kaur, B. H. Choy, J. B. W. Yeo, & S. L. Chin (Eds.), *Excellence in mathematics education: Foundations and pathways. Proceedings of the 43rd annual conference of the Mathematics Education Research Group of Australasia* (pp. 273–280). Singapore: MERGA.

Russo, J, & Hubbard, J. (2022). Changing teacher practices: Differentiating mathematics instruction through sequences of challenging tasks in the early primary years. In N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys. Proceedings of the 44th annual conference of the Mathematics Education Research Group of Australasia* (pp. 16–19). MERGA.

Smith, M., Bill, V., & Sherin, M. G. (2020). *The 5 practices in practice: Successfully orchestrating mathematics discussions in your elementary classroom*. Corwin Press.

Smith, M., & Stein, M. (2018). *5 practices for orchestrating productive mathematical discussion* (2nd ed.). National Council of Teachers of Mathematics.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*(4), 313–340. https://doi.org/10.1080/10986060802229675

Sullivan, P., Bobis, J., Downton, A., Livy, S., McCormick, M., & Russo, J. (2023). *Maths sequences for the early years: Challenging children to reason mathematically.* Oxford University Press.

Sullivan, P., Walker, N., Borek, C., & Rennie, M. (2015). Exploring a structure for mathematics lessons that foster problem solving and reasoning. In V. Geiger, & A. Bennison (Eds.), *Mathematics education in the margins. Proceedings of the 38th annual conference of Mathematics Education Research Group of Australasia* (pp. 41–56). MERGA.

Saleh, A., & Bista, K. (2017). Examining factors impacting online survey response rates in educational research: Perceptions of graduate students. *Journal of MultiDisciplinary Evaluation, 13*, 63–74. https://eric.ed.gov/?id=ED596616

Tomlinson, C. A. (2014). *The differentiated classroom: Responding to the needs of all learners*. ASCD.

Vale, C., Widjaja, W., Doig, B., & Groves, S. (2019). Anticipating students' reasoning and planning prompts in structured problem-solving lessons. *Mathematics Education Research Journal*, *31*(1), 1–25. https://doi.org/10.1007/s13394-018-0239-5

Wallace, A. H. (2007). Anticipating student responses to improve problem solving. *Mathematics Teaching in the Middle School*, *12*(9), 504–511. http://www.jstor.org/stable/41182474