Rethinking the Number Magnitude-Based Progression: An Analysis of Place Value Development in Years 3–6

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Place value is one of the 'big ideas' in number and plays a critical role in helping students develop their number sense, problem solving and computation skills. Yet, the elegant simplicity of our place value system belies the abstract nature of the construct. This paper presents data from 606 Year 3-6 students (ages 8-12) from two metropolitan Melbourne primary schools who completed the Place Value Assessment Tool (PVAT). Each student's place value knowledge was categorised according to the Place Value Developmental Progression (PVDP). The results highlight the wide range of understanding in each year level and challenge the efficacy of a number magnitude-based progression in place value.

In every class within Australia, students are at various points on their journey to understand place value. Some are just beginning to appreciate the idea that 10 ones are 1 ten, while others are confidently able to apply their knowledge to work with decimal place value. It is a teacher's role to determine the level of understanding for each student and decide upon the next best steps. This sounds relatively simple, but most educators who have taught place value would appreciate the considerable challenge this presents.

To effectively teach place value, teachers firstly require access to a quality, research-based assessment tool to determine each student's level of understanding. Next, teachers must determine the instructional tasks which will scaffold a student to take the 'next step' in their development. To do this efficiently a teacher must appreciate the progression students make when coming to understand place value. Across Australia, place value instruction is largely guided by the number magnitude-based progression presented in the current Australian Curriculum document (ACARA, 2023). In the curriculum, students are introduced to 2-digit numbers in Year 1, followed by 3-digit numbers in Year 2 and so on. The aim of this paper is to use student data and the Place Value Developmental Progression (PVDP) created by Rogers (2014) to highlight the potential issues teachers face when using a number magnitude-based progression to guide their instruction. The research question this paper addresses is:

- Do the Australian Curriculum descriptors adequately describe the progression Year 3-6 students make in place value?

Literature

Big Ideas

Research by Siemon and colleagues (2012) identified six 'big ideas' students must obtain to develop mastery in number. The six 'big ideas' are: Trusting the count, place value, multiplicative thinking, partitioning, proportional reasoning, and generalising. Siemon et al. (2012) contend that a focus on these 'big ideas' in number 'strips back' the often overwhelming mathematics curriculum to the 'non-negotiables'. Hurst & Hurrell (2014) describe how the 'big ideas' encourage teachers to take a more global view of mathematics education, allowing them to feel confident to teach students wherever they are in their development. Charles (2005) defines a 'big idea' as a "statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" (p.10). Yet within most mathematics curriculum documents it is not immediately apparent to teachers which content is central to student learning. There is no indication of the relevant importance of individual descriptors. Curriculum descriptors related to the 'big ideas' such as place value, require weeks if not months of targeted instruction (Rogers, 2014),

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whereas other descriptors can be addressed in just a few lessons. In Version 9.0 of the Australian Mathematics Curriculum (ACARA, 2023), the Year 3 syllabus has 23 curriculum descriptors. Only two of these descriptors explicitly mention place value (AC9M3N01 and AC9M3N03). For teachers who do not fully appreciate the importance of place value, the relatively small number of descriptors may lead them to underestimate the depth of understanding and time required to fully address this critical concept. Using the 'big ideas' to guide instruction removes the 'guess work' for teachers.

Place Value: A Big Idea

Place value understanding underpins almost every part of the mathematics curriculum. Counting, estimating, money, addition, subtraction, multiplication, division, converting units and percentage all relate to place value. A lack of understanding in place value has been shown to negatively impact a student's sense of number (McIntosh et al., 1992), understanding of decimals (Moloney & Stacey, 1997) and comprehension of multi-digit operations (Fuson, 1990a, 1990b). Place value is an integral part of the Primary Mathematics Curriculum. Siemon et al., (2012), identified place value as the second 'big idea' in number. Place value is first introduced in the Australian Curriculum in Year 1 and continues to be a focus through to Year 6 (ACARA, 2023). Place value is an abstract concept that takes years to develop. It was described by Major (2011) to be like the framework of a house, supporting students to build further mathematical learning. Siemon (2017) notes that students are at considerable risk of failing to understand the subsequent big ideas (including multiplicative thinking and partitioning) without developing a solid understanding of place value.

Place value can be thought of in two ways: the place value *system* and place value *content*. As adults we are very aware of the place value *system.* This includes understanding the recursive multiplicative base 10 relationship between the place value columns, appreciating the role played by zero, and knowing a digit's value can be determined by its place in the number (Ross, 2002, Silveira, 2021). Yet we cannot simply 'teach' the place value system. We must provide students with multiple opportunities (Department of Education and Training, 2020) to engage with content related to all aspects of place value.

As noted by Major (2011), place value is often an "ill-defined concept in terms of teaching components" (p.16). The Australian Curriculum (v.9.0) uses a variety of verbs throughout Year 1-6 achievement standards and descriptors to describe place value. These include: *partition, rearrange, regroup, rename, recognise, represent* and *order* (ACARA, 2023). Yet these verbs (several of which have very similar meanings) can be easily misinterpreted by teachers. To address this issue, in her doctoral research, Rogers (2014) used Rasch analysis (Rasch, 1960) to empirically show that place value can be broken down into, and defined by, six aspects: *Calculate, Count, Compare/Order, Make/Represent, Name/Record, Rename*. The six aspects provide teachers with structure and clarity around the assessment and teaching of place value.

Assessing the Big Ideas

Using assessment data to guide teaching has been shown to be one of the most effective, empirically proven processes to improve student performance (Black & Wiliam, 1998; Hattie, 2012). Yet not all assessments are created equal. If the items are too easy or too difficult the teachers will gain an incomplete picture of their students' knowledge (Izard, 2002). If online assessments are used, a teacher's involvement in the assessment process is reduced (Rogers, 2021). The opportunity to observe firsthand student responses is lessened, and a greater reliance is placed on a teacher's ability to make inferences from the data generated by the platform (assuming the data collected is valid and reliable) (Popham, 2018). Furthermore, if an assessment does not comprehensively cover the construct it is designed to address, teachers may overlook omitted content and over-emphasise the content included. Rogers (2014) highlighted this through her audit of place value assessments

commonly used in Australian schools. She observed that many assessments failed to include items that addressed all six aspects of place value. For example, the skill of renaming was often overlooked, leading to a lack of teaching and student understanding in this important aspect.

It is critical that teachers have access to comprehensive assessments addressing the 'big ideas'. The Scaffolding Numeracy in the Middle Years (SNMY) project (Siemon et al., 2006), the Reframing Mathematical Futures II (RMFII) project (Siemon et al., 2018) and the place value work conducted by Rogers (2014), have all produced valid and reliable assessments for teachers and associated learning progressions that guide evidence-informed, research-based instruction.

Student Progression in Place Value

Place value is taught in every classroom across Australia, yet very little empirical evidence has been gathered to map student's natural progression through this critical construct. Work by Clements and Sarama (2009) has shown the value of teachers using their knowledge of learning trajectories or developmental progressions to guide instruction. Common practice within Australian classrooms sees teachers use the curriculum to make decisions around the content they cover in a particular year level. Yet, as noted by Daro et al., (2011) curriculum documents are not typically "deeply rooted in empirical studies of the ways children's thinking and understanding in mathematics actually develop" (p.16) This is evident in Version 9.0 of the Australian Curriculum (ACARA, 2023), which consistent with previous versions of the Australian Curriculum, presents a number magnitude-based place value progression. The curriculum states students should explore 2-digit place value, followed by 3-digit place value, 4-digit place value, 5-digit place value and finally decimal place value.

An alternate progression was developed by Rogers (2014). Rasch analysis (Rasch, 1960) was used to create the Place Value Developmental Progression (PVDP). The PVDP provides teachers with an evidence-based description of the typical 'stages' students move through when coming to understand each of the six aspects of place value. The PVDP stages increase in competence from Stage 1 through to Stage 4 and provide teachers with a brief description of the type of thinking students typically display within each aspect. Importantly the PVDP progression is *not* related to number magnitude, but more to the skills and understandings students display related to the six aspects within place value (Rogers, 2014).

Table 1 shows a summary of the recommended foci Rogers (2014) identified for each stage within the six aspects of place value. These foci inform the analysis presented in the discussion of this paper.

Table 1

Place Value Developmental Progression (PVDP) Teaching Foci (Rogers, 2014)

Aspect	Stage 1	Stage 2	Stage 3	Stage 4
Calculate	Simple addition and multiplication involving tens	Basic calculations-composite units	Calculations-multiples of ten	Conceptual meaning behind multiplication and division involving multiples of ten
Compare/Order	Order numbers up to five digits	Identify 'between'	Compare using composite units	Multiplicative comparison
Count	Before/after, less/more	Bridging over centuples	Link between renaming/ counting	Flexible counting in multiple place value parts

Aspect	Stage 1	Stage 2	Stage 3	Stage 4
Make/Represent	Word/block association for base ten models	Explain base ten blocks	Canonical/non-canonical using proportional/ non-proportional models	Representations using other bases
Name/Record	Read/ write to 100	Read/ write to 1000	Read/write–any magnitude	N/A
Rename	How many tens in one hundred	Recognise composite units	Understand 'altogether'	Identify decimal parts when renaming

Methodology

The data referred to in this paper was gathered from two Catholic Primary schools in metropolitan Melbourne, Australia in 2016. Both schools were considered to have slightly above average levels of educational advantage as defined by their Index of Community Socio-Educational Advantage (ICSEA) value. Both school's NAPLAN data showed their Year 3 and 5 cohorts to be average or slightly higher than the national average in terms of their level of proficiency in Numeracy (ACARA, 2016a; ACARA, 2016b). This suggests the sample of students at both schools could be considered to represent a relatively typical cohort of Australian students. Both schools were keen to measure the place value knowledge of their students, and each Year 3-6 classroom teacher agreed to administer the Place Value Assessment Tool (PVAT) to their class at the beginning of the Australian school year (February) during a regular numeracy session. The PVAT was developed in doctoral research by Rogers (2014). At School A, n=296 Year 3-6 students completed the PVAT, while n=310, Year 3-6 students completed the PVAT at School B.

The PVAT is a paper and pen test which addresses the 'big idea' of place value. The PVAT has two parallel forms—Form A and B which were proven to be valid, reliable and equal in difficulty through Rasch analysis (Rasch, 1960). This paper will only refer to the data gathered from Form A of the PVAT. Form A consists of 58 short answer questions. The dichotomous items address the 6 aspects of place value (Rogers, 2014) and cover a range of difficulty levels. The PVAT items are presented from least to most difficult, and students are encouraged to complete as many questions as they can in 60 minutes. The teachers at School A and School B marked the PVAT in accordance with the marking guide and provided de-identified PVAT data sets to the researcher. These sets included the student's year level, gender and PVAT Form A raw score. Using School A and B's data, the researcher translated each student's PVAT raw score into a corresponding stage on Rogers' (2014) Place Value Developmental Progression (PVDP). The process to develop the PVDP and the raw score translator is explained in much greater detail in Rogers (2014). The results below present the PVDP stages of the Year 3-6 students in Schools A and B determined from their PVAT raw scores.

Results

School A and B's results show a developmental progression through the four PVDP stages and across the year levels (see Figure 1 and 2). This means that in both School A and B, there is a large percentage of Stage 1 students in Year 3 (60% at School A and 30% in School B) but this decreases to almost zero by Year 6. Conversely, in both schools, there are no students in Year 3 at Stage 4, but by Year 6 a substantial number of students have reached this stage (64% in School A and 38% in School B).

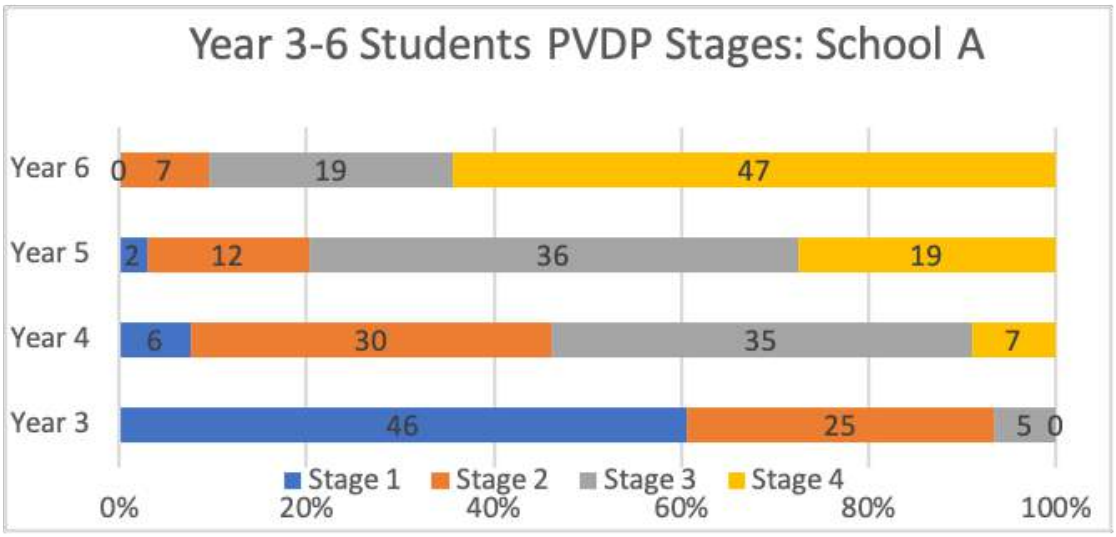

Figure 1. Place value developmental progression stages: School A.

The data also shows that within the eight cohorts of students there is a wide range of place value understanding. For example, Year 4 and 5 in School A and Years 4, 5 and 6 in School B have four stages of development amongst students, whilst Year 3 and 6 in School A, and Year 3 in School B have three stages present.

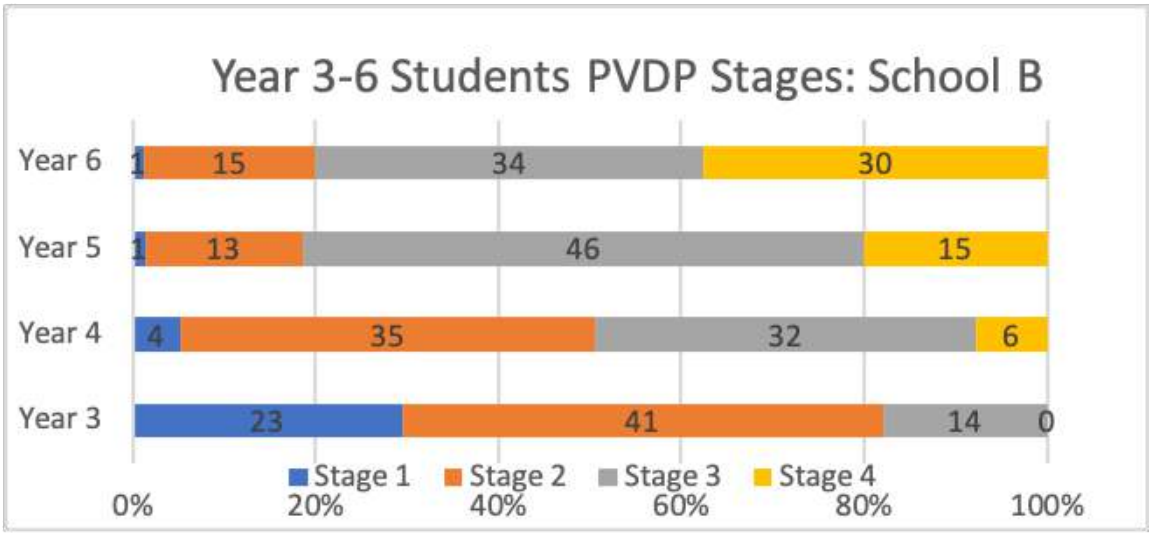

Figure 2. Place value developmental progression stages: School B.

Discussion

The importance of providing instruction within each student's zone of proximal development has been well established in the literature (Clements & Sarama, 2009; Siemon et al., 2012; Vygotsky, 1978). Yet determining exactly what content is within reach of students is a challenge for teachers. For the most part, curriculum standards are informed by, and reflect research related to the progression students make when coming to understand mathematics. However, in the construct of place value, the progression presented in the Australian Curriculum (ACARA, 2023) contrasts with the PVDP developmental progression presented in Table 1. Two examples from the data presented above will be used to illustrate the potential issues faced by teachers using the number magnitude-based progression to guide instruction.

In Year 3, 93% of School A and 82% of School B students were found to be in PVDP Stage 1 or 2. Looking at Version 8.1 of the Australian curriculum, Year 3 students in 2016 (when this data was gathered) were required to: "Recognise, model, represent and order numbers to at least 10 000 (ACMNA052)" (ACARA, 2016). According to the PVDP foci presented in Table 1, in the aspect of 'compare/order', Stage 1 and 2 students would benefit from ordering 5-digit numbers and identifying the number 'between' two numbers. It would, therefore, be appropriate for these students to complete tasks involving ordering numbers of the magnitude suggested in the Year 3 curriculum descriptor (to at least 10,000). However, looking at the other five aspects within PVDP in Stages 1 and 2, it is not developmentally appropriate for students to count, calculate, rename, read, write or represent numbers of this magnitude. If a Year 3 teacher in School A or B was to provide place value instruction across these five place value aspects using 5-digit numbers or beyond, over 80% of the students in each class would fail to have the knowledge required to successfully engage with this content.

Looking at the data presented in Figure 1 and 2 we can see that a large proportion of Year 4 students at School A (91%) and School B (92%) display Stage 1-3 understanding in place value. The Australian Curriculum version 8.1 (ACARA, 2016) required decimals to be introduced to students in Year 4. Yet the Stage 1-3 PVDP foci presented in Table 1 suggest these students require work exclusively on whole number place value. It is therefore unrealistic and counterproductive to introduce decimal place value to students in PVDP Stages 1-3. As noted by Moloney and Stacey (1997), the concept of decimals relies on successfully integrating a thorough knowledge of the whole number place value system with the decimal system. The PVDP suggests Stage 1-3 students have not yet mastered whole number place value and introducing them to decimal place value places unnecessary pressure on both them and their teachers. This leads to superficial teaching and disengaged students. It is important to note that by PVDP Stage 4, students are considered developmentally ready to move to decimal place value. Thus, in School A and B, the introduction of decimals would be a more appropriate curriculum standard for the Year 6 cohort.

Implications: Teacher Education

The two examples above show the importance of teachers being aware of the PVDP so as to refine their teaching of place value and better address the needs of their students. Place value is made up of six separate but interconnected aspects (Rogers, 2014). Each aspect requires a distinctive teaching and learning 'cadence'. For example, the rename aspect requires a sophisticated level of thinking underpinned by an appreciation of abstract composite units (Steffe et al., 1983). Renaming is multiplicative in nature, and multiplicative thinking has been shown to develop slowly in students. The PVDP indicates that renaming instruction needs to be slow and deep across the four stages. In contrast, ordering numbers appears in Stage 1 of the PVDP and is not considered a cognitively demanding skill. Being able to order 5-digit numbers does not indicate mastery in place value, yet success with a task involving numbers of this magnitude may provide teachers with an inflated opinion of a student's place value understanding. This is particularly true if the teachers are following a number magnitude-based progression, such as Australian Curriculum.

As Bednarz and Janvier (1982) noted, Year 3 and 4 children can easily compare numbers using a *digit-by-digit* procedure-based method. This means, just as placing words in alphabetical order does not require comprehension of the word's meaning, ordering numbers can be achieved without an appreciation of quantity. It is important for teachers to understand that ordering is a superficial place value skill that requires only a small amount of instruction time, while renaming is a complex skill requiring much more time. High-quality, strategic in-service and pre-service teacher education is required to ensure teachers understand the nuance required within each aspect. This knowledge will help teachers to see place value as a construct made up of smaller skills that must be taught at different rates to ensure success.

Implications: Australian Curriculum Revision

Since the data presented in this paper was gathered, ACARA has released several updates of the Australian Curriculum. The most recent and significant update, Version 9.0, was released in 2022. It should be noted that Version 9.0 continues to present a number magnitude-based place value progression. While the author acknowledges the need for data to be gathered from current Year 3-6 classrooms to further validate its conclusions, it advocates for a revision of the place value descriptors in Version 9.0 of the Australian Curriculum. Currently one Year 3 descriptor states: "recognise, represent and order natural numbers using naming and writing conventions for numerals beyond 10 000 (AC9M3N01)" (ACARA, 2023). This descriptor requires students to work with numbers well beyond the capabilities of most Year 3 students at School A and B. It also fails to acknowledge the distinctive nature of the six aspects within place value. Similarly in Year 4, students are expected to explore decimal place value. Descriptor AC9M4N01 states: "recognise and extend the application of place value to tenths and hundredths and use the conventions of decimal notation to name and represent decimals." (ACARA, 2023). Again, this expectation was observed to be well beyond the ability of most Year 4 students at School A and B. Both these examples highlight the significant revision required to ensure the Australian Curriculum more closely reflects the six aspects of place value and the research-based PVDP progression. These revisions will assist teachers to be more accurately informed when identifying the next 'best step' for each student, and are an important first step in improving the teaching and learning of place value across Australia.

Conclusion

This paper used the place value assessment data gathered from eight Year 3-6 classrooms and the Place Value Developmental Progression (PVDP) to emphasise three key points: the importance of teachers understanding student progression in place value through quality teacher education, the distinct role each of the six aspects play in this progression, and the necessity to reconsider the current number magnitude-based place value progression used in Version 9.0 of the Australian Curriculum.

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