A Curriculum Comparison of Years 9–10 Measurement and Geometry in Australia and Singapore

Ellen Sugianto
University of South Australia
ellen.sugianto@mymail.unisa.edu.au

South Australia's PISA performance has been in constant decline since 2003 with the proportion of PISA participants meeting the Australian national proficient standard dropping from 73% (in 2003) to 50% (in 2018). In contrast, Singapore is a consistently strong performer. To better understand student readiness in answering PISA questions, this paper reports a curriculum comparison between Australia and Singapore for Years 9 and 10 in Measurement and Geometry. The findings highlight the similarities and differences in the topics covered in both countries' curricula and raise questions about potential implications for student outcomes in PISA.

The Programme for International Student Assessment (PISA) is a triennial assessment conducted by the Organisation for Economic Co-Operation and Development (OECD). PISA is targeted at 15 year olds as students at this age are reaching the end of compulsory education in most of the participating countries (OECD, 2013a). PISA attempts to assess how well students are able to apply what they have learnt from school in unfamiliar, real-world contexts. The PISA mathematical literacy assessment assesses four content categories: Change and Relationships, Space and Shape, Quantity, Uncertainty and Data. The outcomes are presented as mean scores, distributions of scores, and percentages of participants who attain defined proficiency levels (Thomson et al., 2013).

In the Measurement Framework for Schooling in Australia 2020, the national proficient standard for 15-year-old students participating is Level 3 on the PISA scales (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2020). Students who attain national proficient standard demonstrate that they have acquired more than the elementary skills expected at that year level. As such, the proportion of participating students achieving at or above the national proficient standard is a key performance measure.

South Australia (SA)'s PISA performance in mathematics has been reported as in constant decline since 2003. The 2018 PISA data indicated that SA's mean score was in the bottom three states in Australia (Thomson et al., 2019). Additionally, of SA students who participated in PISA, the proportion meeting the national proficient standard was only 50% in 2018 in contrast to 73% in 2003. This is concerning as SA data suggests a substantial increase in the proportion of students not meeting the national proficient standard. Across the four content categories in 2012, SA students recorded the lowest scores in Space and Shape (Thomson et al., 2013). Amongst the top three performers in 2012 in Space and Shape, Singapore attained a mean score of 580 points (Thomson et al., 2013). In comparison, SA's was 481 points.

This paper comprises six sections. First, it outlines what constitutes mathematical proficiency in PISA. Next, a case is made for the concept on *intended curriculum*. The third section reviews the curriculum structure of Australia and Singapore. This is followed by the methodology used in this paper. The comparative findings are then presented to give a highlight the differences and similarities of coverage of Measurement and Geometry in both curricula. Finally, the paper concludes with the potential implications for student readiness in answering PISA assessment items in Space and Shape content category.

Mathematical Proficiency

The PISA 2012 framework identifies three mathematical processes and seven capabilities. Mathematical processes describe "what individuals do to connect the context of a problem with

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mathematics and thus solve the problem” (OECD, 2013a, p. 28). The three processes are:(1) formulating situations mathematically, (2) employing mathematical concepts, facts, procedures, and reasoning, and (3) interpreting, applying and evaluating mathematical outcomes (OECD, 2013a). Each of the mathematical processes is underpinned by mathematical capabilities. Mathematical capabilities are learnable and complement mathematical content to be engaged with to solve PISA assessment items. The seven capabilities are (1) communication, (2) mathematising, (3) representation, (4) reasoning and argument, (5) devising strategies for solving problems, (6) using symbolic, formal and technical language and operations, and (7) using mathematical tools (OECD, 2013a).

Drawing on the mathematical processes and capabilities, the OECD formulated six increasing levels of mathematical proficiency. The proficiencies are intended to describe a series of mathematical capabilities required to solve PISA assessment items from each level. The difference in the activation and/or complexity level of mathematical capabilities across the mathematical processes is the key to describing the proficiency level (Stacey & Turner, 2015). For example, the description for Level 1 (see Figure 1) describes students as being able to carry out routine mathematical procedures with explicit direction in Level 1, while Level 6 describes students as being able to apply and use the aspects of all seven mathematical capabilities.

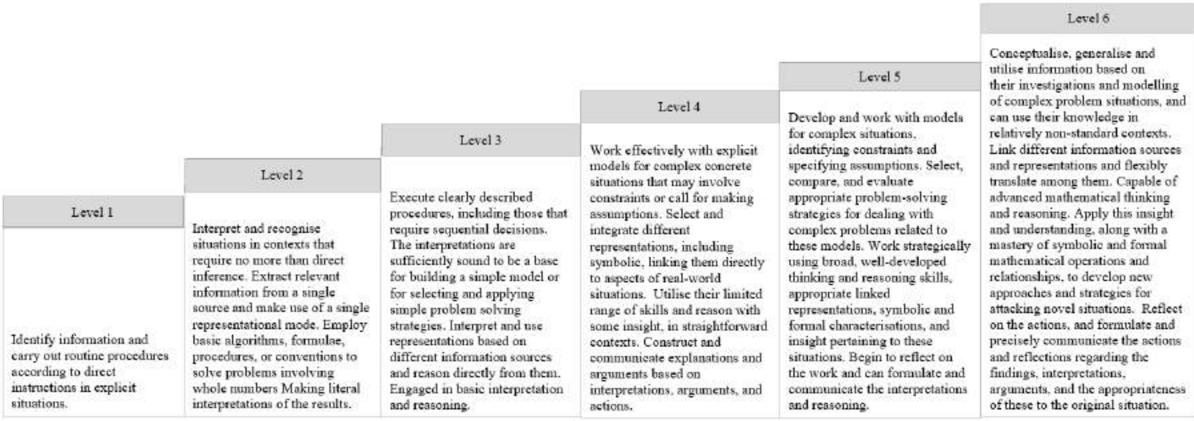

Figure 1. PISA 2012 mathematics proficiency levels (OECD, 2013b, p. 61).

The mathematical capabilities also require different levels of complexity to be activated. For example, in the communication capability, the lowest complexity level is in assessment items that simply require a numeric answer. Items requiring a justification or explanation have a greater level of communication complexity (OECD, 2013a). Figure 1 provides an overview of the difference in the proficiency level from Level 1 (the lowest) to Level 6 (the highest) (OECD, 2013b, p. 61) —the description of each proficiency level is intended to characterise the knowledge and skills of students at the level (Thomson et al., 2013; Turner, 2014).

Intended Curriculum

Schmidt et al. (2004) argue that the differences in achievement among countries are related to what is taught, suggesting “the curriculum itself makes a huge difference" (pp.2–3). For example, students in a country where the education system has a greater emphasis on Geometry would perform better in the assessment items in the Space and Shape category of PISA (OECD, 2013b). Hypotheses like this have led researchers to compare curricula of specific countries against those from top-performing countries.

Studies such as Safrudiannur and Rott (2019), and Acar and Serçe (2021) have focused on curriculum comparisons, with curriculum defined to be the intended curriculum. Others have taken a broader approach to curriculum. Valverde et al. (2002) modify a tripartite model curriculum by adding the potentially implemented curriculum (mediating role of textbooks) into the intended curriculum (the educational system aims and goals), the implemented curriculum (the enactment of these goals in teaching and learning) and the attained curriculum (what students attained from the teaching and learning). In the comparison of mathematics achievement across countries, Valverde et al. (2002) created "a powerful link between the intended and the implemented curricula in their creation of the potentially implemented curriculum, affected primarily by the textbook" (O'Keeffe, 2013, p.3). Further, Valverde et al. (2002) explained that a key function of mathematics textbooks is to turn abstract curriculum policy into more concrete instructions for teachers and students; textbooks transform the intention of curricular policy into instructions in the classrooms. Therefore, mathematics textbooks can be seen as representative of curriculum (Remillard, 2005).

The larger study from which this paper stems analyses both the intended and potentially implemented curriculum layers related to the Year 9 and 10 Measurement and Geometry strands for South Australia and Singapore. Singapore is chosen for curriculum comparison based on their continual high performance in PISA. By conducting a curriculum comparison of the Years 9 and 10 Measurement and Geometry strands, followed by a content analysis of the mathematics textbooks, this larger study aims to inform the discussion of student readiness in answering PISA questions in the Space and Shape content category and to provide a clearer picture of the proficiency needed to solve tasks presented in the mathematics textbooks.

This paper presents the first phase of the study, which is a comparison of the Australian Curriculum: Mathematics (AC:M) Years 9 and 10 Measurement and Geometry strand and the Singaporean Express course for both O-Level Mathematics and O-Level Additional Mathematics (SC:M).

Curriculum Structure in Australia and Singapore

The AC:M is a national curriculum for mathematics for Foundation to Year 10 and is adopted without modification in South Australia. In addition to the Year 10 curriculum is a Year 10A curriculum which caters for students who are seeking an extension in Year 10. The focus of this study is on Years 9–10A Measurement and Geometry strand in AC:M version 8.3 as the students who participated in the latest PISA in 2018 would have used mathematics textbooks written for this version of the curriculum. The AC:M is structured around the interconnection between three strands and four proficiency strands. The content strands, which describe what to be taught and learnt, are Number and Algebra, Measurement and Geometry, and Statistics and Probability. The proficiency strands, which describe how the content is explored, include understanding, fluency, problem solving, and reasoning. In Years 9–10A, the Measurement and Geometry strand is divided into three sub-strands: using units of measurement, geometric reasoning, and Pythagoras and trigonometry (ACARA, 2016).

The Singaporean mathematics curriculum (SC:M) consists of a set of connected syllabuses to cater to students' interests and strengths. There are five mathematics syllabuses in the secondary mathematics curriculum which are tied to the three core courses that are designed to match students' academic progress and interest such as Express course, Normal (Academic) course and Normal Technical course (Ministry of Education, 2019). The highest percentage of secondary school student enrolment in 2018 was those students undertaking the Express stream, which was about 63% (Ministry of Education, 2019). The Express course offers O-level Mathematics and O-level Additional Mathematics which assumes knowledge of O-level Mathematics content in addition to more in-depth coverage of topics (Ministry of Education, 2012a, 2012b). SC:M combines Secondary 3 and Secondary 4 O-Level Mathematics content strands into one syllabus (Ministry of Education, 2012b) and similarly for Secondary 3 and Secondary 4 O-Level Additional Mathematics (Ministry of Education, 2012a). Hence, the Express course pertaining to Measurement and Geometry in O-

Level Mathematics syllabus, and Geometry and Trigonometry in O-Level Additional Mathematics syllabus in Secondary 3 and 4 were selected for this paper.

Research Design

Mayring's qualitative content analysis (Mayring, 2014) was used to inform the research design. Notably, Singapore was also selected by ACARA (2018) in their recent comparative curriculum study, which compared the Australian Curriculum against the Singapore Curriculum as part of a regular study of international comparison of Australia with high-performing countries. Of relevance to this study is the mathematics curriculum comparison for Years 9 and 10, which was based on the same version of the curricula analysed in this paper. The ACARA comparison comprised three layers of analysis: breadth, depth, and rigour of the content descriptions and elaborations in the AC:M against the content descriptions and learning experiences in the comparable Singaporean curriculum. For breadth analysis, ACARA counted the total number of content descriptions and elaborations for the AC:M, and content descriptions and learning experiences for the Singaporean curriculum, in addition to noting down content not present in the AC:M.

This study employs a similar approach to the analysis undertaken by ACARA (2018) however a point of difference is the "unit" that is counted. The ACARA (2018) analysis used the content description as the "unit", with each description counted as one "topic". In this study the detail provided in each content description is refined by using the elaborations to identify the number of single "topics" across the strand of measurement and geometry. In other words, one description could result in more than one topic being identified. For example, consider the content description ACMMG221 "Solve problems using ratio and scale factors in similar figures". The content description alone does not specify if the ratio and scale factors are used for corresponding sides or for areas of similar figures, however the elaboration "establishing the relationship between areas of similar figures and the ratio of corresponding sides (scale factor)" provides additional detail. The outcomes were two topics: a topic named "ratio and scale factors of the corresponding sides in similar figures" and a topic named "ratio of areas of similar figures".

All content descriptions from the AC:M Years 9–10A strand of Measurement and Geometry were examined to identify topics for inclusion in this study. This list of topics was then used as the basis for comparison, meaning that topics from the AC:M were used as a reference and SC:M topics mapped onto this. Finally, topics in SC:M Sec 3/4 Measurement and Geometry that are not in the AC:M were also identified.

Findings

This section presents the findings of the curriculum comparison for the AC:M and SC:M Years 9-10A Measurement and Geometry strand. Once the topics included in the AC:M Measurement & Geometry strand for Years 9–10A and SC:M Secondary 3/4 were identified, the next step was to identify the topics that were common (or not) to both curricula. This mapping resulted in 52 topics, which can be categorised as shown in Figure 2.

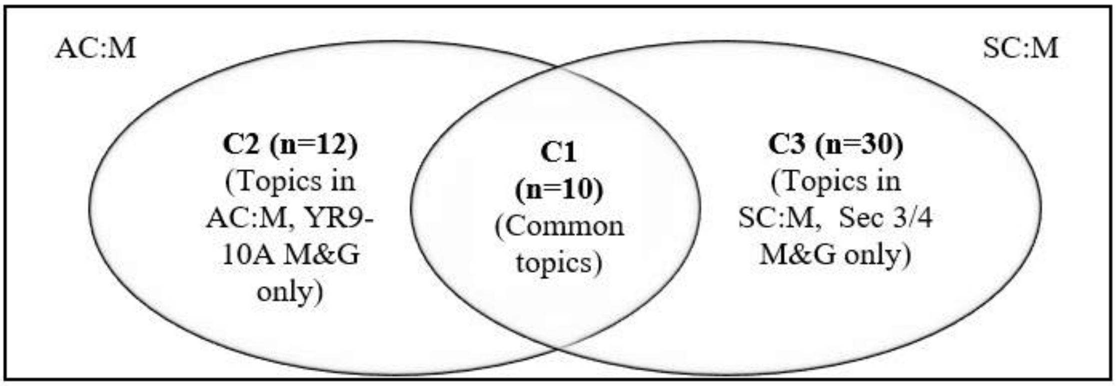

Figure 2. Classification used for topics in AC:M and SC:M Measurement & Geometry.

Table 1 lists the ten topics in C1, accounting for 19% of the topics identified in this analysis. Table 2 lists the twelve topics in C2. Eleven of these topics appear in AC:M Year 9, 10 or 10A Measurement and Geometry but appear in earlier year levels in the SC:M. These topics account for 21% of the total. However, it is worth noting that these eleven topics account for 50% of all topics identified in AC:M Years 9-10A Measurement and Geometry. The twelfth topic, "very small and very large time scales and intervals", is located in the comparable year level in SC:M but in the Number and Algebra strand.

Table 1

C1: Topics in AC:M Years 9–10A Measurement and Geometry and SC:M Secondary 3/4 Measurement and Geometry

In the sub-strand: Geometric reasoning	In the sub-strand: Pythagoras and trigonometry
Ratio of areas of similar figures	Solve simple trigonometric equations
Formulate proofs involving congruent triangles and angle properties	Solve right-angled triangle problems including those involving direction and angles of elevation and depression
Use congruence and similarity to proof and numerical exercises involving plane shapes	Sine, cosine and area rules for any triangle and solve related problems
Angle and chord properties of circles	Apply Pythagoras' Theorem and trigonometry to solve three-dimensional problems
Use of properties of geometric figures	Unit circle to define trigonometric functions and graph them with and without the use of digital technologies (sine & cosine functions)

Table 2

C2: Topics in AC:M Years 9–10A Measurement and Geometry, but not in SC:M Secondary 3/4 Measurement and Geometry

In the sub-strand: Using units of measurement	In the sub-strand: Geometric reasoning	In the sub-strand: Pythagoras and trigonometry
Area of composite shapes (rectangles and triangles)	Enlargement and condition of similar triangle	Pythagoras' Theorem involving right-angled triangles
Surface area and volume of cylinders	Scale diagram	Use similarity to investigate the constancy of sine, cosine and tangent ratio in right-angles triangles
Surface area and volume of right prisms	Ratio and scale factors of the corresponding sides in similar figures	Apply trigonometry to solve right-angle triangle problems
Surface area and volume of prisms, cylinders and composite solids		
Surface area and volume of right pyramids, right cones, spheres and related composite solids		

Table 3 lists 7 (of 30) topics in C3, which are from SC:M 3/4 Measurement and Geometry and in a different strand of the AC:M Years 9–10A curricula (in Number and Algebra). Table 4 lists the remaining 23 topics in C3, which are from SC:M Secondary 3/4 Measurement and Geometry but do not appear in the AC:M Years 9–10A curricula, presumably because they are covered in the senior years. These topics account for 44% of the total.

Table 3

C3: Topics in SC:M Secondary 3/4 Measurement and Geometry, and in a Different Strand in AC:M Years 9–10A

In the strand: Number and Algebra	
Gradient of a line segment on the Cartesian plane	Solve problems involving parallel and perpendicular lines
Distance between two points on the Cartesian plane	Midpoint of a line segment on the Cartesian plane
Sketch linear graphs using the coordinates of two points	Describe, interpret and sketch circles
Solve linear equations from graphs	

Table 4

C3: Topics in SC:M Secondary 3/4 Measurement and Geometry, but not in AC:M Years 9–10A

Topics not in AC:M Years 9–10A		
Arc length, sector area and area of segment of a circle	Unit circle to define trigonometric functions and graphs of y= tan(x)	Vectors in two dimensions (use of notation)
Use of radian measure of angle (including conversion between radians and degrees)	Other trigonometric functions for angles of any magnitude (secant, cosecant and cotangent)	Representing a vector as a directed line segment
Perpendicular and angle bisector	Principal values of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$	Translation by a vector
Ratio of volumes of similar solids	Unit of angles of trigonometric functions in radians	Position vectors
Mid-point theorem	Use of trigonometric identities, for example cosA/sinA = cot A; expansions of sin (A±B), cos (A±B), tan (A±B), formulae of sin2A, cos 2A and tan2A; expression of acosθ + bsinθ in the form Rcos (θ±α) or Rsin (θ±α)	Magnitude of a vector
Graphs of parabolas with equations in the form $y^2 = kx$	Simplification of trigonometric expressions	Use of sum and difference of two vectors to express given vectors in terms of two coplanar vectors
Area of rectilinear figure	Proofs of simple trigonometric identities	Multiplication of a vector by a scalar
Transformation of given relationships, including $y=ax^n$ and $y=kb^x$, to linear form to determine the unknown constants from a straight-line graph		Geometric problems involving the use of vectors

Summary and Conclusion

The findings show that there is a clear difference between the Measurement and Geometry strand of the intended mathematics curriculum for AC:M Years 9–10 AC:M and SC:M Secondary 3/4. There are more topics ‘not present’ (44%) in AC:M Years 9–10A Measurement and Geometry compared to topics that are ‘common’ in both curricula (19%). Topics that appear prior to Secondary 3/4 in Singapore account for 21% of the total number of topics, and the remaining proportion relate to topics that appear in a different strand in the comparable years in AC:M and SC:M (in Number and Algebra strand). Therefore, these findings imply that students in Years 9 and 10 in South Australia have less coverage of Measurement and Geometry content compared with students in Singapore, which may mean that students in Singapore are better prepared for solving PISA assessment items. The curriculum comparison by ACARA (2018) reported a similar observation.

The intended mathematics curriculum is one factor, of many, that contribute to students’ preparedness to achieve the national proficient standard in PISA mathematics. Other factors include previous outcomes and experiences, depth of knowledge in the topics being assessed, and confidence and capacity in problem solving. Nonetheless, the intended curriculum of each country, as a mandatory curriculum, is a common factor within that country. Hence, the intended curriculum is a useful unit of measure to provide insight into the differences in the content coverage in the Australian and Singaporean curricula in Measurement and Geometry.

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