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The Reification of the Array: The Case of Multi-Digit Multiplication

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Mathematical representations play an important role in the development of understanding. Studies have shown the array to be a powerful representation that allows access to the important theoretical constructs of multiplication, as it highlights composite units and how they build on each other to produce a whole. While there has been substantial research on the array with single-digit multiplication, there exists limited research on its use in multi-digit multiplication.

The focus of this research concerned students' changing use of the array as a *representation of* a contextual situation through to a *representation for* mathematical reasoning over the course of an instructional sequence. Gravemeijer (1999) described this change in use as a process of reification, where the representation takes on object-like character through student activity. Gravemeijer presents two levels in the reification process: the referential level (contextually bound student activity) and the general level (context independent student reasoning).

Design Research methods were employed for the study, which included a teaching experiment component. The experiment comprised four key teaching episodes taught over 10 one-hour lessons and was conducted in two separate Year 5 classes in Sydney with a combined sample size of 45 students. Student work samples, transcribed video recordings, and field notes formed the data collected. Saxe's (2004) form-function framework was used to observe how students' use of the array shifted over the course of the instructional sequence, to serve differing functions. The *form* of the array was defined as specific visual features, and its *function* was defined as the way the students chose to interact with the array in their work.

In the first and second teaching episodes, students used a form of the array that was closely connected to the context of the problem. In the same way, their interactions with the array were contextually bound. The function of the array in this form was to support calculations. Students use of the array was indicative of the *referential* level in the reification process. By the final teaching episode, students' reasoning with the array was more abstract and generalised and removed from the context of the problem; they had progressed to the *general* level in the reification process.

An interim level in the reification process was observed in the third teaching episode, which I termed *structuring*. In the third teaching episode, students were faced with their own misconceptions as they were asked to solve problems using an area-model form of the array. In the process of making sense, student activity was removed from the context of the problem as they explored the multiplicative structure. Significantly, students regressed to a simple grid array in the process of sense making. Central to the structuring level was the flexibility for students to select and use different forms of the array, recognising that different forms will serve different student-generated functions.

References

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