Mathematics Education Research Group of Australasia
MERGA 44: Quick Reads
MERGA 44 Mathematical Confluences and Journeys
Launceston, 3-7 July 2022

"It has the same numbers, just in different order": Middle School Students Noticing Algebraic Structures Within Equivalent Equations

Jodie Hunter
Massey University
<j.hunter@massey.ac.nz>

Jodie Miller
The University of Queensland
<Jodie.miller@uq@edu.au>

Alex Bowmar
Massey University
<a.bowman@massey.ac.nz>

Ian Jones
Loughborough University
<i.jone@lboro.acu.uk>

In both curriculum documents and research studies (Schifter, 2018), there has been increasing emphasis placed on early algebra and the need to facilitate students to work flexibly with numbers and notice relationships and mathematical structure. However, in the teaching of arithmetic, it appears that students are rarely given the opportunities to focus on and build an appreciation of mathematical structures when forming generalisations, and instead utilise calculation to recognise patterns and mathematical structures (Arcavi et al., 2017).

In this paper, we explore student solutions to a free response mathematical assessment task which had opportunities for students to notice structural properties in the context of number systems. We aimed to answer the following research questions: (i) *what do students notice in a task involving algebraic structures with equivalent equations?* And (ii) *how do students notice and explain structural properties and relationships in the context of number systems?*

In total, 308 students aged between 10 years to 13 years participated in the study. The students were given a free-response task involving a set of equations and follow-up prompts to position them to notice, describe, explain, and generalise the structural properties without the need for calculation. The set of equations was designed to include matching pairs which could be identified through noticing structural properties and relationships in the context of number (e.g., $76 \times 15 =$; $(70 \times 5) + (70 \times 10) + (6 \times 10) + (6 \times 5) =$). The students' responses were coded to determine whether they noticed algebraic structures or relationships in the task. Then, the coded responses were re-analysed for the pairs of equations students identified, whether the student used calculation or relational strategies, and the students' explanations.

Findings indicate that students were able to recognise equation pairs that drew on both the associative and distributive properties. Furthermore, the sophistication of students' explanations varied from no explanations given, showing how they solved the equations, using informal language (e.g., "the same numbers, they are just jumbled up."), and providing general examples of the structure (e.g., "they are the same numbers but in a different way, so it doesn't matter like $11 + 10 = 21$ and $10 + 11 = 21$"). Implications from this study include that there needs to be a greater focus on supporting students to notice the structure of number properties across different equations and then to use this to form generalities.

References

Arcavi, A., Drijvers, P., & Stacey, K. (2017). *The learning and teaching of algebra: Ideas, insights, and activities*. Routledge.

Schifter, D. (2018). Early algebra as analysis of structure: A focus on operations. In: Kieran, C. (Ed.). *Teaching and learning algebraic thinking with 5- to 12-year-olds.* Springer.